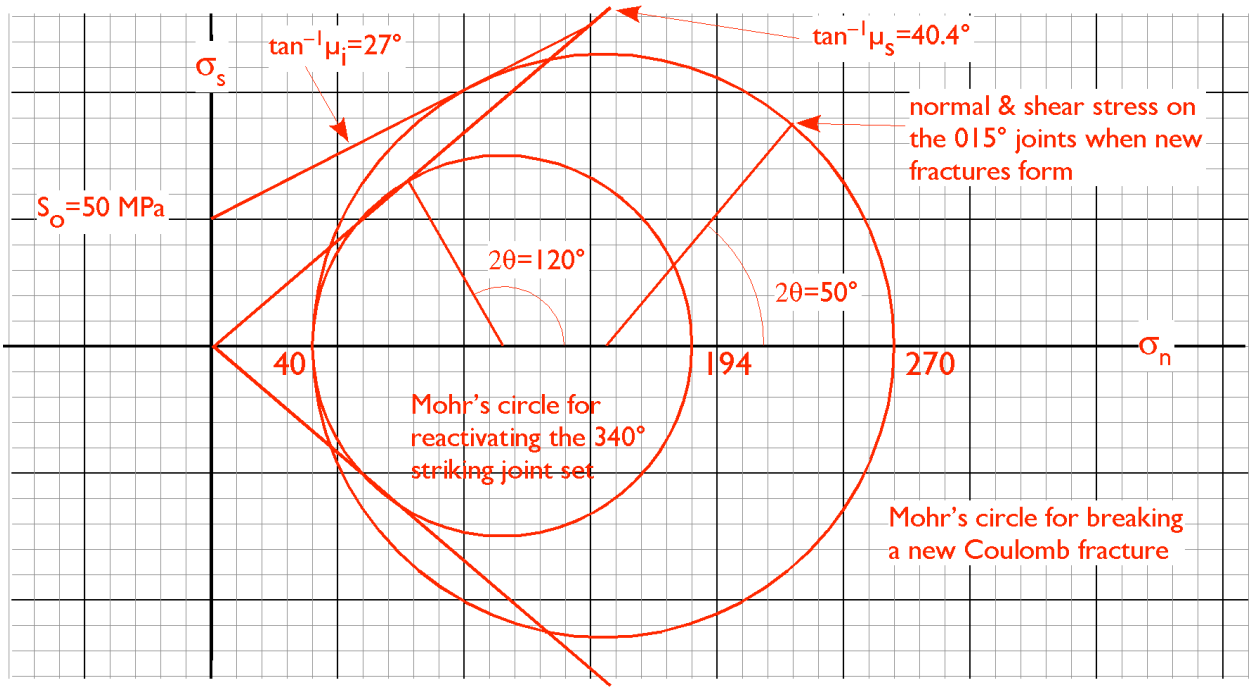
EAS 326-06 MIDTERM EXAM

This exam is closed book and closed notes. It is worth 150 points; the value of each question is shown at the end of each question. At the end of the exam, you will find two pages of potentially useful equations.

Density, ρ =	2720 kg m^{-1}
Cohesion, S_o =	50 Mpa
Coefficient of internal friction, μ_i =	0.51
Coefficient of sliding friction, μ_s =	0.85

1. The Ithaca area has two well developed sets of vertical fractures (joints) with strikes of 340° and 015°. The rocks have the mechanical properties shown in the table at the right. Assume that σ_1 is horizontal, and that the confining pressure is equal the the lithostatic load.

   a. What is the maximum possible magnitude of σ_1 that these rocks can support at 1500 m depth? Show your calculations and plot your results as a Mohr's Circle for stress on the graph paper, below. Be sure to label the axes. [20 pts.]



It was perfectly okay to solve this problem graphically, but here is how to solve it analytically. To get the maximum possible value of sigma 1:

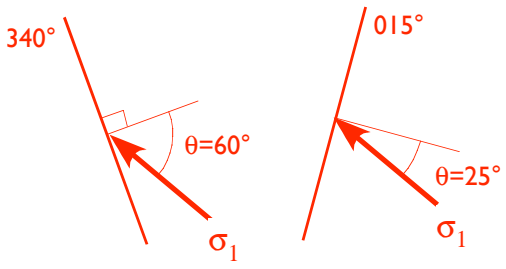
$$\sigma_1 = C_o + K\sigma_3; \quad K = \frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\sin 27}{1-\sin 27} = 2.665; \text{ and } C_o = 2S_o\sqrt{K} = 2(50MPa)\sqrt{2.665} = 163.3MPa$$

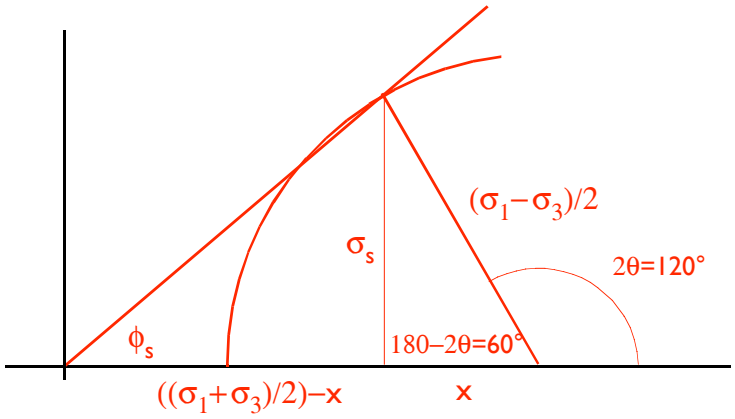
$$\sigma_3 = \rho g z = \left(2720 kg m^{-3}\right)\left(9.8 ms^{-2}\right)(1500m) = 39984000 Pa = 40 Mpa$$

$$\sigma_1 = 163.3MPa + 2.665\sigma_3 = 163.3MPa + 2.665(40MPa) = 270MPa$$

b. If σ_1 is oriented horizontally with a compass azimuth of 130°, will the maximum value be reached? If not, what will happen to prevent it from reaching it's maximum value. Explain your answer with reference to the Mohr's Circle in part (a). [15 pts.]

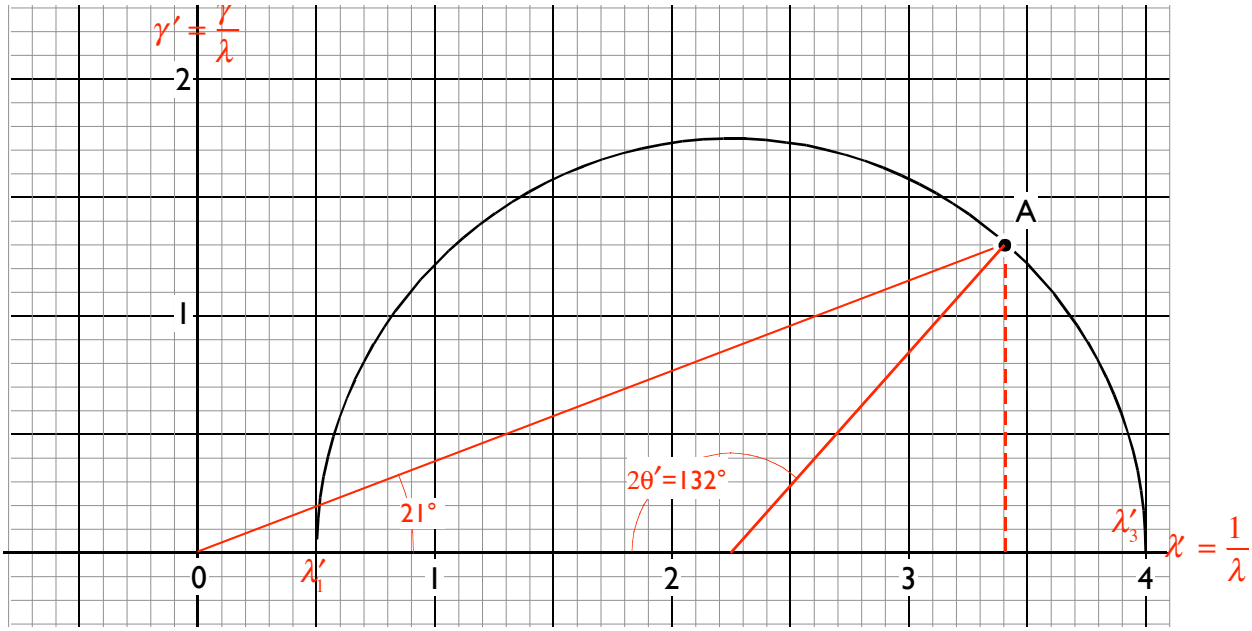
The geometry of the two joints with respect to σ_1 is shown below and the positions of the planes of the two joints is plotted on the Mohrs Circle on the previous page. The 015° joint set will not be reactivated, but the 340° set will be reactivated. You can solve for σ_1 when slip begins on the 340° set graphically, or you can derive it analytically. The latter is slightly complicated because the radius of the Mohr's Circle is not perpendicular to the failure envelop.





$$\sigma_1 = \sigma_3\left(\frac{\mu_s\left(1+\cos\left(180-2\theta\right)\right)+\sin\left(180-2\theta\right)}{\sin\left(180-2\theta\right)+\mu_s\left(\cos\left(180-2\theta\right)-1\right)}\right) = 194\,MPa$$

2. The graph paper below shows a Mohr's Circle for finite strain.



a. Label the axes and intercepts. [5 pts.]

b. What is the *extension* and the *shear strain* experienced by the line represented by point A? What is the orientation of that line with respect to the principal axes of the strain ellipse? [15 pts.]

From the Mohrs Circle for strain, above:

$$\lambda' = 3.4 \quad \therefore \quad \lambda = \frac{1}{3.4} = 0.294; \quad S = \sqrt{\lambda} = 0.542; \quad e = S - 1 = -0.458$$

$$\gamma = \tan\psi = \tan 21 = 0.384$$

The line makes and angle of 66° with respect to the maximum principal axis of finite strain.

c. Assuming plane strain, is the deformation volume constant or not? Show how you determined your answer. [15 pts.]

No, the deformation is not volume constant because the two principal stretches (or quadratic elongations) are not the reciprocals of one another. Assuming that λ'_3 is correct, λ'_1 should equal 0.24 (i.e., 1/4) if the deformation were volume constant.

d. Of the deformation mechanisms that we have discussed in class, which one(s) might be able to explain the deformation and why? Explain your answer and describe the types of features in the rock that you might look for to confirm your answer. [15 pts.]

Pressure solution could produce the above deformation by dissolving away the rock in the principal shortening direction. You might look for stylolites, truncated fossils, etc. To identify the presence of pressure solution.

3. Two lines have orientations: 347, 62 and 243, 21. Calculate the angle between them. [15 pts.]

This problem is most easily solved using the dot product of two vectors. First, calculate the direction cosines for each line:

	Line A	*Line B*
cos α	cos(347)cos(62) = 0.457	cos(243)cos(21) = -0.424
cos β	sin(347)cos(62) = -0.106	sin(243)cos(21) = -0.832
cos γ	sin(62) = 0.883	sin(21) = 0.358

$$\theta = \cos^{-1}\left(\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2\right)$$
$$= \cos^{-1}\left[(0.457)(-0.424) + (-0.106)(-0.832) + (0.883)(0.358)\right] = 77.85°$$

4. Accurately and concisely define/explain the following terms: [50 pts]

a. Power law creep

Power law creep is a flow law that describes how a rock will deform via dislocation glide and climb under different temperature, strain rate and differential stress conditions. This type of deformation is very sensitive to temperature, as shown by the presence of T in the equation, below:

$$\dot{\varepsilon} = C_o\left(\sigma_1 - \sigma_3\right)^n \exp\left(\frac{-Q}{RT}\right)$$

b. Point defects

There are two basic types of point defects: (1) vacancies, and (2) impurity atoms. The latter can be divided into (2a) substitution or (2b) interstitial impurities depending on the atomic radius of the impurity atom relative to the crystal lattice. While impurity atoms can hinder deformation at relatively low temperatures by pinning dislocations (cold working or strain hardening), the increase in temperature increases the overall number of vacancies which can facilitate deformation by allowing dislocations to climb over/around the impurity atoms.

c. Viscoelastic

Viscoelastic deformation is non-permanent but time dependent deformation. When stress is applied, the material accrues strain not instantaneously but over a finite amount of time. Likewise, when stress is removed the deformation is recovered over a finite amount of time.

d. Hydrofracturing

The pressure of confined fluid in the pores of a material acts is just the opposite way as confining pressure. The pore fluid pressure only affects the normal stresses, not the shear stresses (because of the spherical stress state in a fluid. Thus, while an increase in confining pressure makes a material stronger, an increase in pore fluid pressure makes a material weaker. In terms of the Mohrs Circle for stress, an increase in pore fluid pressure does not change the size of the circle (i.e., the differential stress), but does shift the circle towards the origin. If the pore fluid pressure is sufficiently high, it can shift the circle far enough towards the origin that it intersects the failure envelope, and the material experiences "hydrofracturing." This trait is commonly used to help extract fluids from the ground (hydrocarbons, or water).

e. Non-coaxial deformation

Non-coaxial deformation is that in which the principal axes of strain at each instant in time are NOT parallel to the principal axes in the previous step in the deformation. Such deformation is commonly referred to as simple shear.

Potentially Useful Equations

Note that not all of these equations are needed for the exam and that some of them have not, or will not, be covered in class.

$$\sigma_{\Delta T} = \frac{\alpha E \Delta T}{1-\upsilon}$$

$$\sigma_{ij} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$

$$V_i = \frac{k_{ij}}{\eta}\left(\frac{dP_P}{dx_j}\right)$$

$$\sigma_n^* = \left(\frac{\sigma_1^* + \sigma_3^*}{2}\right) + \left(\frac{\sigma_1^* - \sigma_3^*}{2}\right)\cos 2\theta$$

$$\sigma_s = \left(\frac{\sigma_1^* - \sigma_3^*}{2}\right)\sin 2\theta$$

$$\lambda' = \left(\frac{\lambda_3' + \lambda_1'}{2}\right) - \left(\frac{\lambda_3' - \lambda_1'}{2}\right)\cos 2\theta'$$

$$\gamma' = \left(\frac{\lambda_3' - \lambda_1'}{2}\right)\sin 2\theta'$$

$$\tan\theta' = \tan\theta\frac{\sqrt{\lambda_3}}{\sqrt{\lambda_1}} = \tan\theta\frac{S_3}{S_1}$$

$$\sigma_s = S_o + \sigma_n^*\mu$$

$$\dot{\varepsilon} = C_o(\sigma_1 - \sigma_3)^n \exp\left(\frac{-Q}{RT}\right)$$

$$\dot{\varepsilon} = C_o(T)\frac{D(\sigma_1 - \sigma_3)}{d^n}$$

$$U = -\frac{C_1}{r} + \frac{C_2}{r^{12}}$$

$$P_{lith} = \int_0^z \rho g dz$$

$$\phi = \frac{V_f}{V_f + V_s}$$

$$\phi = \phi_o \exp(-az)$$

$$\Delta_v = \frac{V_{final} - V_{initial}}{V_{initial}}$$

$$e = \frac{l_f - l_i}{l_i}$$

$$e = \frac{\sin(\phi + \theta)}{\sin\phi} - 1$$

$$S = \frac{l_f}{l_i} = \sqrt{\lambda}$$

$$\lambda = S^2$$

$$\lambda' = \frac{1}{\lambda}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta = \left(\frac{1+\cos 2\theta}{2}\right)$$

$$\sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$U_i = U_{o_i} + E_{ij}dx_j$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} U_{o_1} \\ U_{o_2} \\ U_{o_3} \end{bmatrix} + \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix}$$

$$\sigma_m^* = \frac{\left(\sigma_1 + \sigma_2 + \sigma_3 - 3P_f\right)}{3}$$

$$\sigma_1^* = C_o + K\sigma_3^*$$

$$K = \frac{1+\sin\phi}{1-\sin\phi} \;;\quad C_o = 2S_o\sqrt{K}$$

$$\sigma_\tau = 0.85\sigma_n^*$$

$$\sigma_\tau = 50\text{ MPa} + 0.6\sigma_n^*$$

$$(\alpha+\beta) = \frac{(1-\lambda_f)\mu_f + \beta}{(1-\lambda)k+1}$$

R = 8.3144 x 10^{-3} kJ/mol °K
= 1.9872 x 10^{-3} kcal/mol °K

°K = °C + 273.16°

1 MPa = 106 kg/m s^2 = 10 bars

g = 9.8 m/s^2 = 980 cm/s^2

$$\cos\alpha = \cos(trend)\cos(plunge)$$
$$\cos\beta = \sin(trend)\cos(plunge)$$
$$\cos\gamma = \sin(plunge)$$

$$\cos\alpha = \sin(strike)\sin(dip)$$
$$\cos\beta = -\cos(strike)\sin(dip)$$
$$\cos\gamma = \cos(dip)$$

$$\tan 2\theta' = \frac{2}{\gamma}$$

$$p_i = \sigma_{ij} l_j$$

$$p_1 = \sigma_{11}l_1 + \sigma_{12}l_2 + \sigma_{13}l_3$$
$$p_2 = \sigma_{21}l_1 + \sigma_{22}l_2 + \sigma_{23}l_3$$
$$p_3 = \sigma_{31}l_1 + \sigma_{32}l_2 + \sigma_{33}l_3$$

$$L_d = 2\pi T_3\sqrt{\frac{E}{6E_o}}$$

$$L_d = 2\pi T_3\sqrt{\frac{\eta(S-1)}{6\eta_o(2S^2)}}$$

$$C \equiv \frac{1}{r}$$

$$C_G = C_{\max}C_{\min}$$

$$\beta = \theta - \phi + (180° - 2\gamma)$$

$$\phi = \tan^{-1}\left\{\frac{-\sin(\gamma-\theta)\left[\sin(2\gamma-\theta)-\sin\theta\right]}{\cos(\gamma-\theta)\left[\sin(2\gamma-\theta)-\sin\theta\right]-\sin\gamma}\right\}$$

$$\phi = \theta = \tan^{-1}\left\{\frac{\sin(2\gamma)}{2\cos^2(\gamma)-1}\right\}$$

$$\gamma = \tan\psi = 2\tan\left(\frac{\delta}{2}\right)$$

$$\gamma = \tan\psi \cong 0.0175(\delta)$$

$$\bar{s} = 2\bar{h}\tan\left(\frac{\delta}{2}\right)$$

$$\bar{s} \cong 0.0175\bar{h}(\delta)$$

$\Sigma\Delta M = 0 = \Delta M_w + \Delta M_s + \Delta M_c + \Delta M_m + \Delta M_a$

$0 = \Delta(\rho_w h_w) + \Delta(\rho_s h_s) + \Delta(\rho_c h_c) + \Delta(\rho_m h_m) + \Delta(\rho_a h_a)$

$\Delta E = \Delta h_w + \Delta h_s + \Delta h_c + \Delta h_m + \Delta h_a$

$$\theta = \cos^{-1}\left(\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2\right)$$